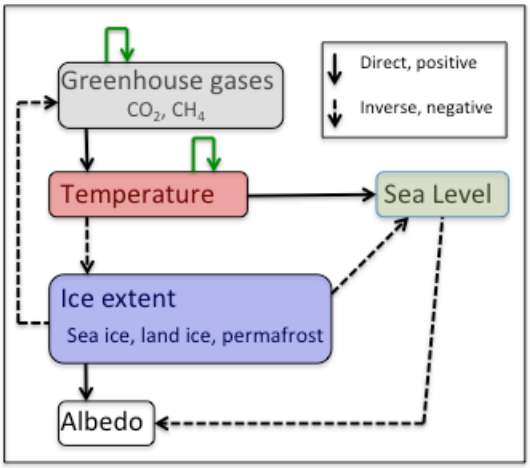
Stability Analysis of the Climate System

The climate system is represented as shown below, where the green arrows represent possible external factors that could positively or negatively influence greenhouse gases or temperature.



The system can be translated into an interaction matrix where each cell entry a_{ij} represents the positive, negative, or negligible (0) effect of the row component on the column component:

	Greenhouse Gases	Temperature	Ice	Albedo	Sea level
GHG	$\pm a_{11}$	a_{12}	0	0	0
Temp	0	$\pm a_{22}$	$-a_{23}$	0	a_{25}
Ice	$-a_{31}$	0	0	0	$-a_{35}$
Albedo	0	$-a_{42}$	0	0	0
SL	0	0	0	$-a_{54}$	0

Feedback at level *k* represents the mutual influence of system components on each other for loops with *k* components: $Fk = \Sigma(-1)^{m+1} Z(m, k)$ where the latter term is the product of *m* disjunct loops with *k* components. The Routh-Hurwitz criteria are necessary and sufficient conditions for stability; thus violation indicates dynamical instability. They are that $Fk < 0$, for all *k*, and that successive Hurwitz determinants are positive. A more complete description of this type of analysis is available in Chapter 2 of this masterpiece.

The feedbacks for the system here are as follows.

$F1 = (\pm a_{11}) + (\pm a_{22})$

$F2 = (\pm a_{11})(\pm a_{22})$

$F3 = a_{12}(-a_{23})(-a_{31}) + (-a_{23})a_{34}(-a_{42}) + a_{25}(-a_{54})(-a_{42})$

$F4 = [(-a_{54)}(-a_{42})(-a_{23})(-a_{35})] - [(-a_{23})a_{34}(-a_{42})(\pm a_{11})]$

$F5 = - [(-a_{54)}(-a_{42})(-a_{23})(-a_{35})(\pm a_{11})]$

F1, F2 may be positive or negative, depending on the signs of a_{11}, a_{22} and, if different in the case of F1, their relative magnitude. F4, F5 could conceivably be negative, depending on the sign of a_{11} and the relative strength of the two loops shown. However, F3 > 0, violating the RHC and indicating an unstable system.